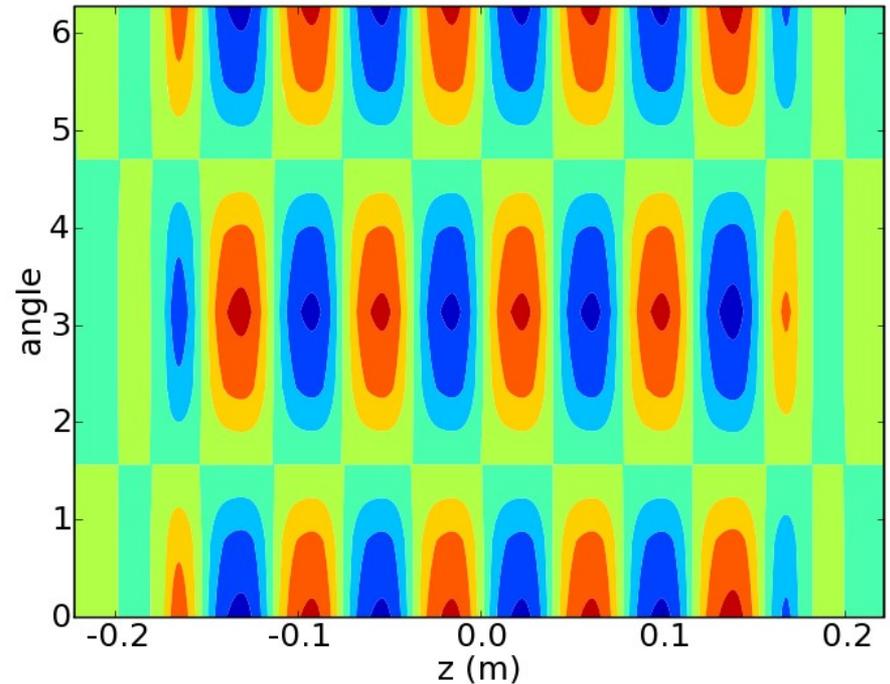
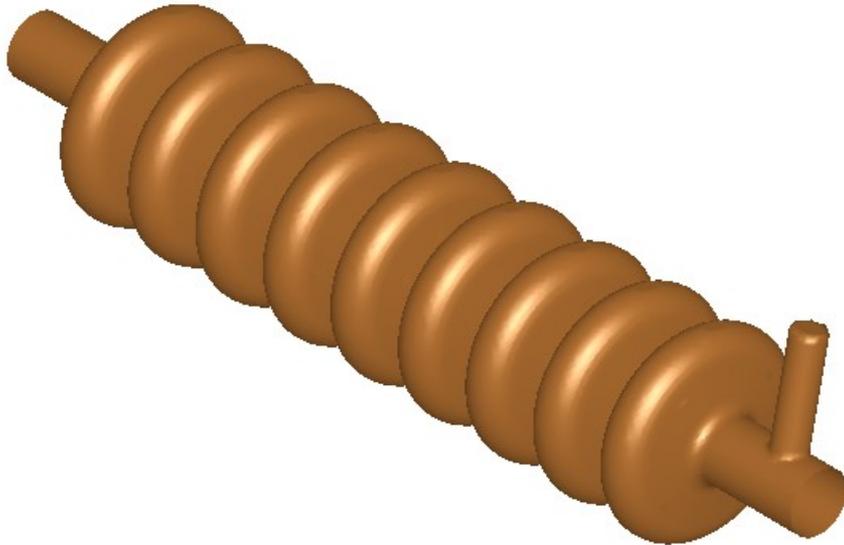




# Coupling ComPASS EM codes with Beam Dynamics codes



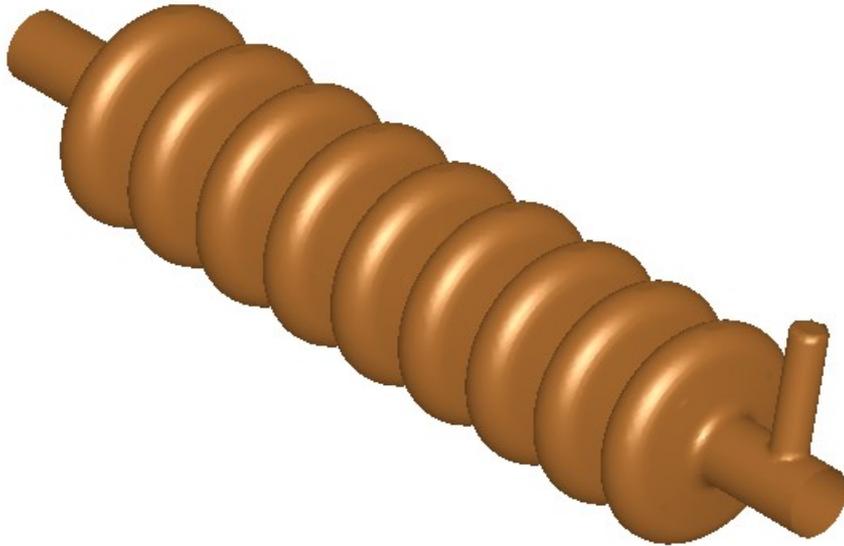
P. Stoltz, I. Pogorelov, D. Abell

Tech-X Corporation

J. Amundson

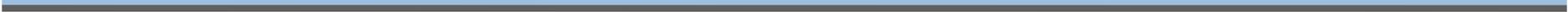
Fermi National Laboratory

**Problem: detailed EM simulations are great, but small-scale, and accelerators are really big**

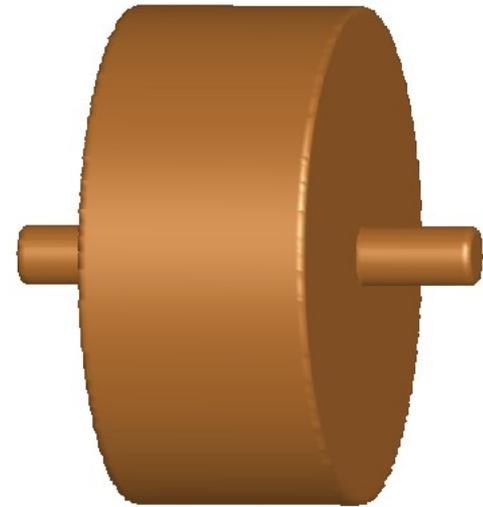
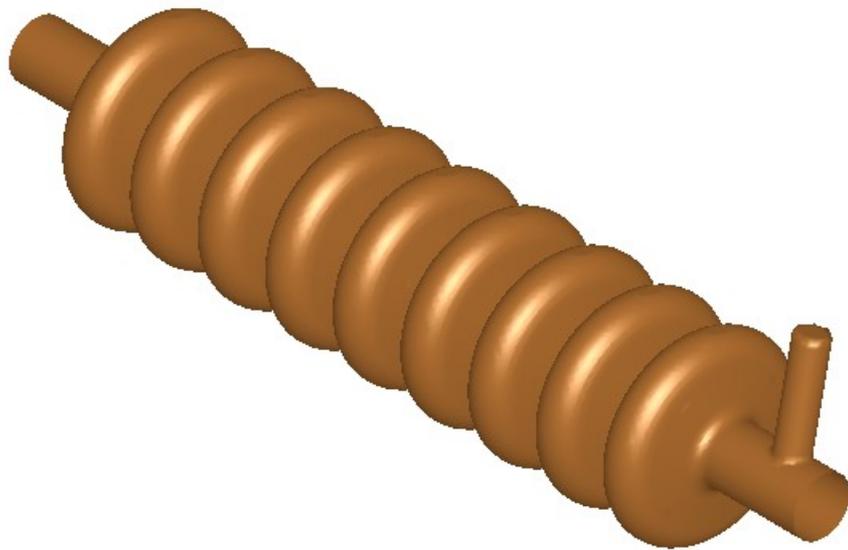


EM simulations: cm-m

FNAL: km



**Problem: Particle tracking codes are great, but lack the detail of EM simulations**



VORPAL, OMEGA3P

MaryLie-Impact, Synergia

# Solution: Combine the detail of EM codes with the large-scale capability of tracking codes



PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS **9**, 052001 (2006)

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## Numerical computation of high-order transfer maps for rf cavities

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(Received 18 November 2005; published 9 May 2006)

Modern map-based accelerator beam-dynamics codes model magnetic elements so as to include nonlinear effects and realistic fringe fields, but they persist in modeling rf cavities as either energy kicks or linear maps. This work presents a method for including the nonlinear effects of rf cavities in a map-based code.

DOI: [10.1103/PhysRevSTAB.9.052001](https://doi.org/10.1103/PhysRevSTAB.9.052001)

PACS numbers: 41.20.Jb, 41.75.-i, 41.85.Ja

The techniques in this paper allow us to make maps for particle tracking codes from EM simulations using FDTD/FEM codes.

Good for SciDAC: code coupling, multiscale physics, etc

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Math is not difficult: Fourier decompose in  $\phi$  and  $z$ ,  
and decompose with Bessel functions in  $r$



$$E_z(R, \phi, z) = E_{zc0}(R, z) + \sum_{m=1}^{\infty} [E_{zcm}(R, z) \cos(m\phi) + E_{zsm}(R, z) \sin(m\phi)].$$

$$\tilde{e}_m(k) = \frac{1}{R_m(k, R)} \int_{-\infty}^{\infty} \frac{dz}{\sqrt{2\pi}} e^{-ikz} E_{zcm}(R, z)$$

Assumes harmonic time dependence: codes like Omega3P are ideal,  
and codes like VORPAL can use new mode extraction techniques

Radial dependence comes from Maxwell's equations

---

The advantage of this approach is one can then analytically expand about the axis



$$E_{\rho cm}(\rho, z) = \sum_{j=0}^{\infty} \frac{(\rho/2)^{m-1+2j}}{j!(m+j)!} C_{\rho cmj}(z),$$

$$E_{\phi cm}(\rho, z) = \sum_{j=0}^{\infty} \frac{(\rho/2)^{m-1+2j}}{j!(m+j)!} C_{\phi cmj}(z),$$

$$E_{zcm}(\rho, z) = \sum_{j=0}^{\infty} \frac{(\rho/2)^{m+2j}}{j!(m+j)!} C_{zcmj}(z),$$

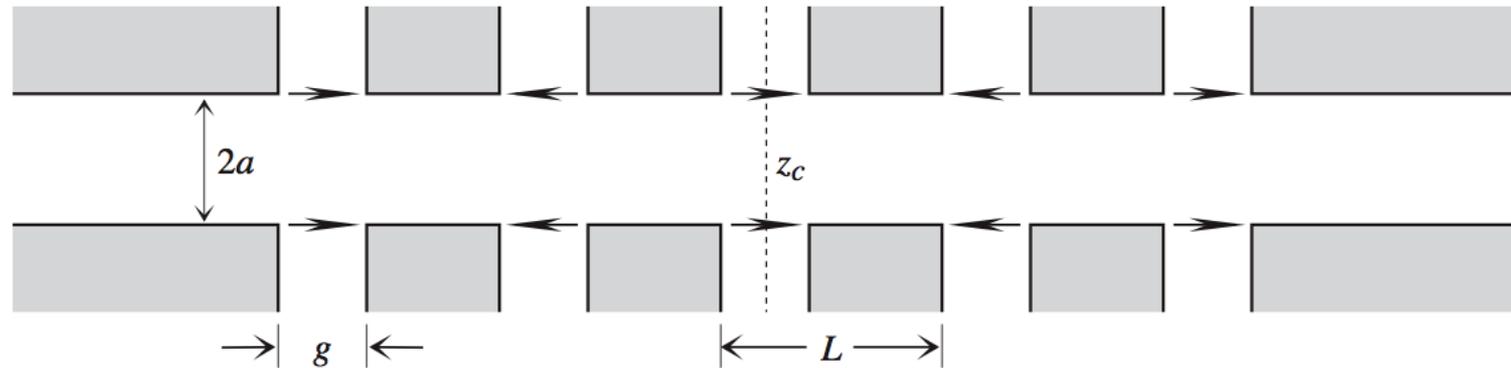
$$C_{\rho cmj}(z) = \int_{-\infty}^{\infty} \frac{dk}{\sqrt{2\pi}} e^{ikz} \left( \frac{-ik}{\kappa_l} \right) s_l(k)^j \kappa_l^{m-1+2j} \\ \times \left( j s_l(k) \tilde{e}_m(k) + \frac{1}{2} \tilde{\beta}_m(k) \right),$$

$$C_{\phi cmj}(z) = \int_{-\infty}^{\infty} \frac{dk}{\sqrt{2\pi}} e^{ikz} \left( \frac{ik}{\kappa_l} \right) s_l(k)^j \kappa_l^{m-1+2j} \\ \times \left( j s_l(k) \tilde{f}_m(k) - \frac{m+2j}{2m} \tilde{\alpha}_m(k) \right),$$

$$C_{zcmj}(z) = \int_{-\infty}^{\infty} \frac{dk}{\sqrt{2\pi}} e^{ikz} s_l(k)^j \kappa_l^{m+2j} \tilde{e}_m(k).$$

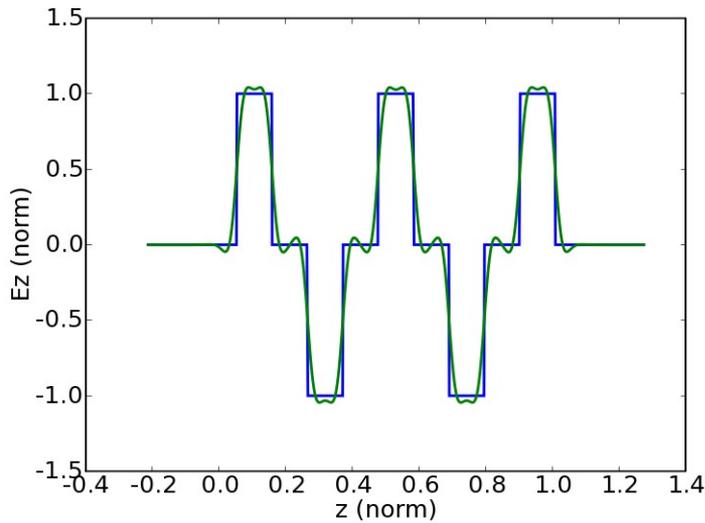
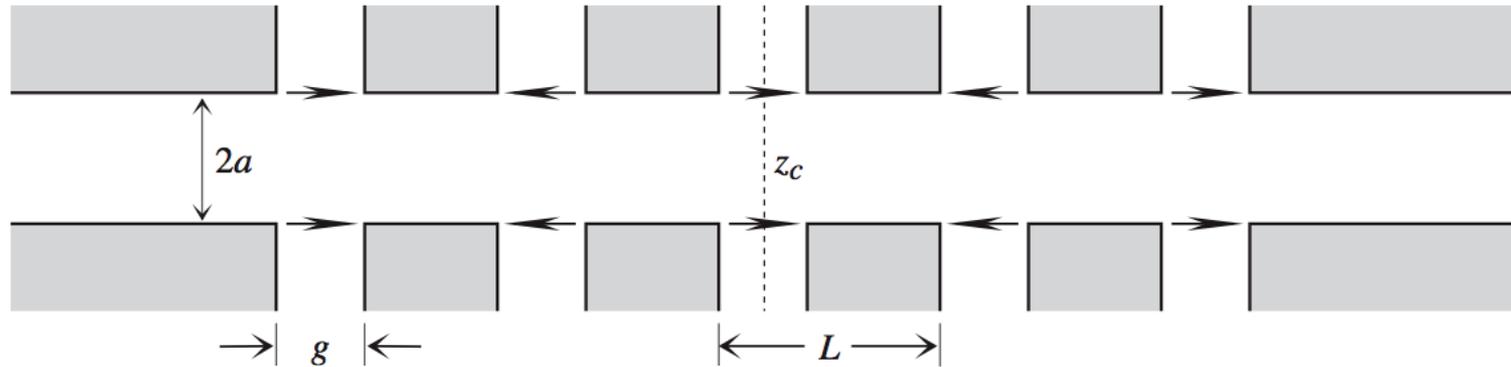
Analytic expansions give smooth derivatives (important for making maps)

Recipe: get simulation data at the largest radius possible, use this data to create maps for all smaller radii

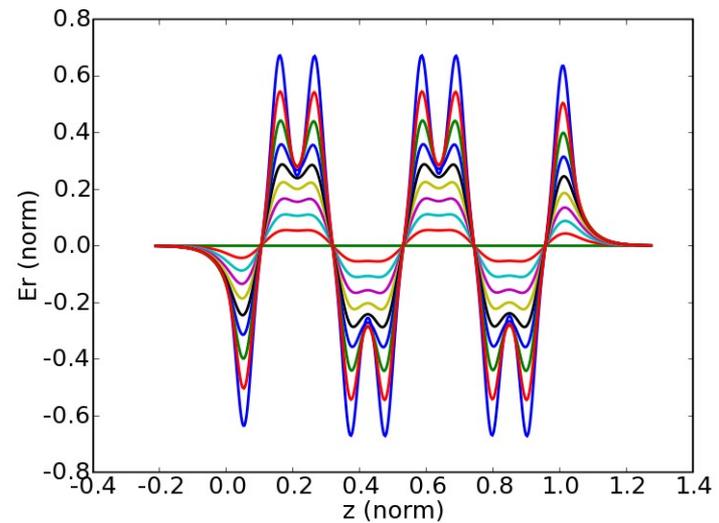


- Use VORPAL/OMEGA3P to generate detailed EM models of rf cavities
- Get lineout data at the largest possible radius (typically the pipe radius) at a few azimuthal positions
- Abell technique will generate analytic approximations to the field for all radii inside the chosen radius. Use TPSA to create maps from the fields.
- Use the maps in your favorite tracking code (ML/I, Synergia, etc)!

# Ex. 1: We can match analytic results of a simplified five-cell accelerating structure to a few percent

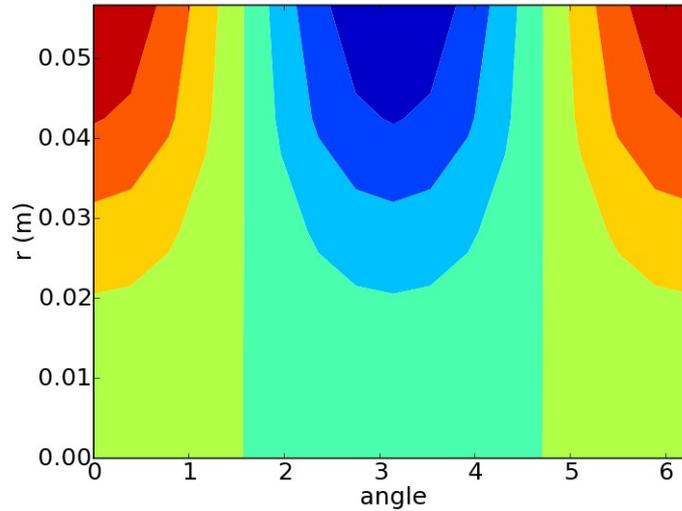
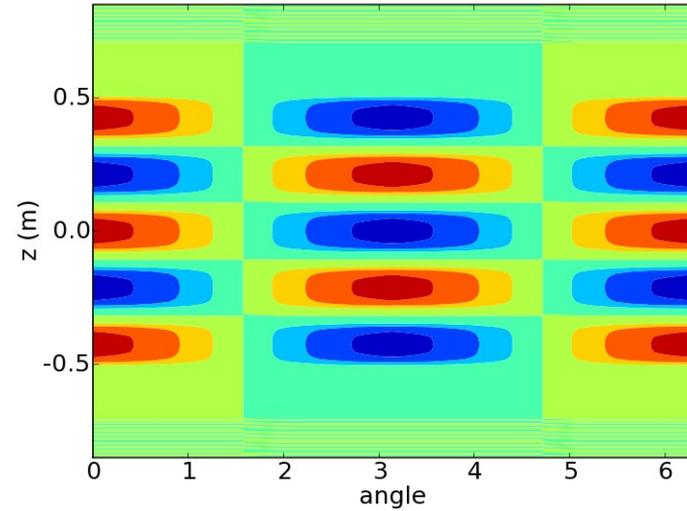
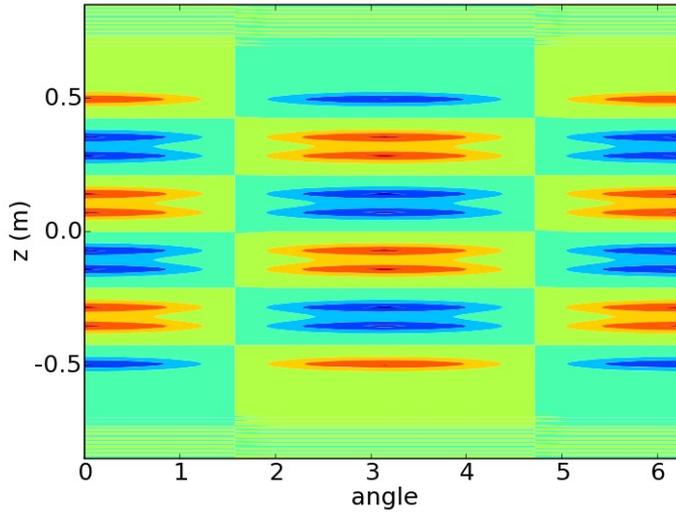
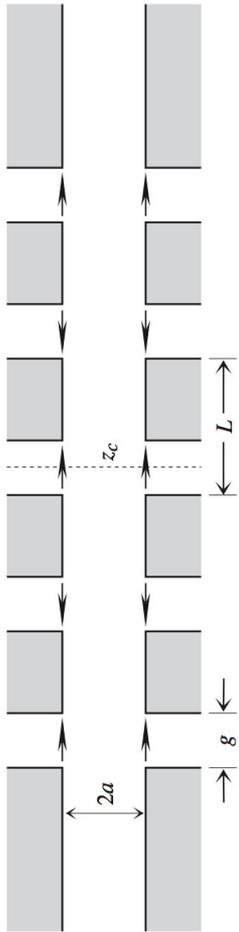


Analytic (blue) and Abell model (green) for  $E_z$

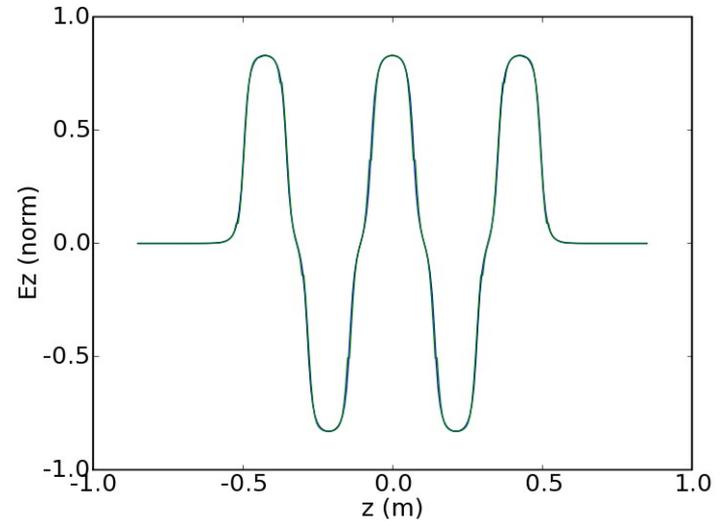
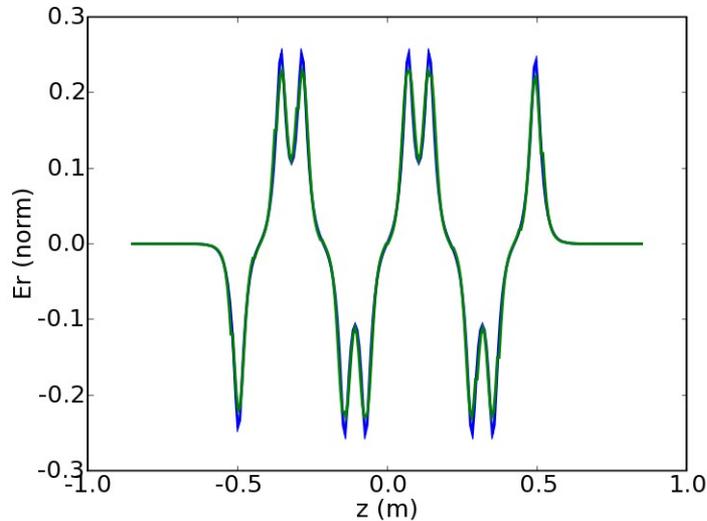


Abell model for  $E_r$  at various radii

Ex 2: We can also match a crab ( $m=1$ ) mode to within a few percent

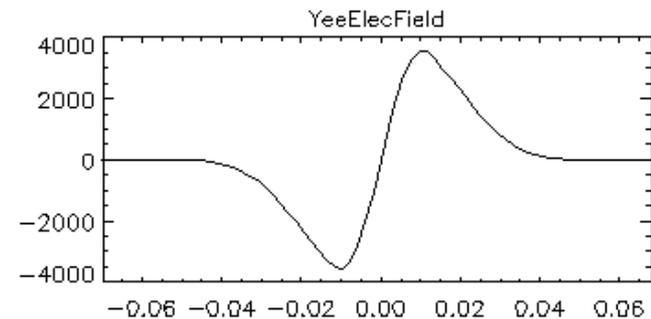
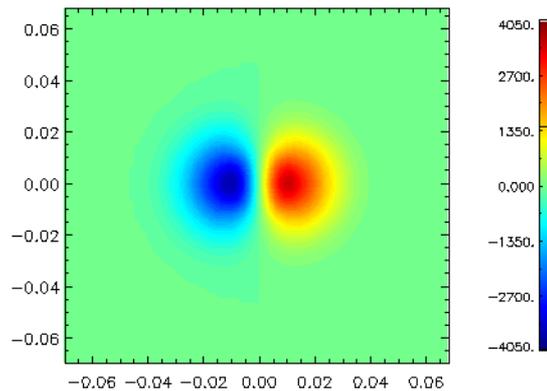
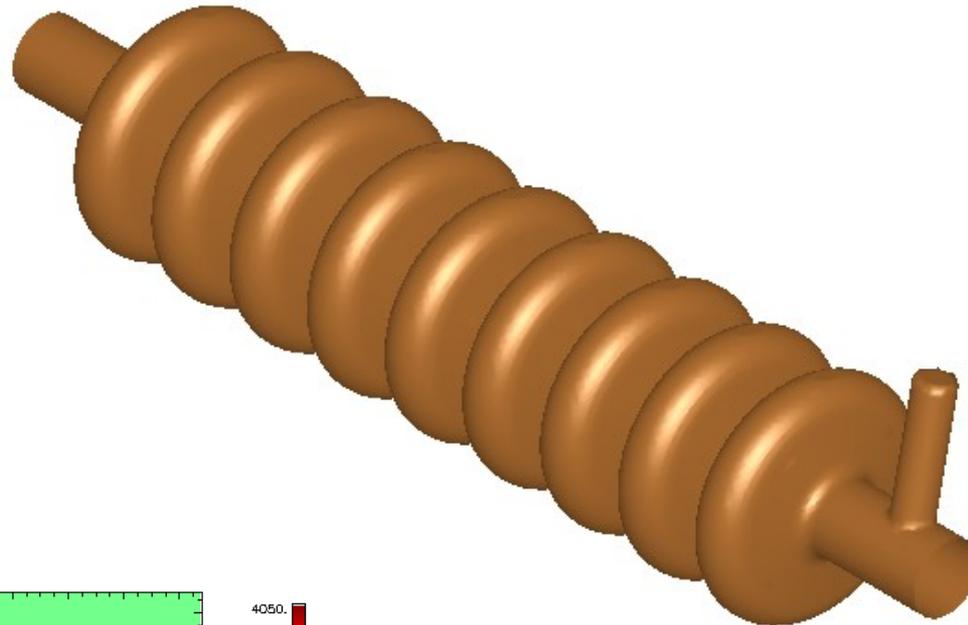


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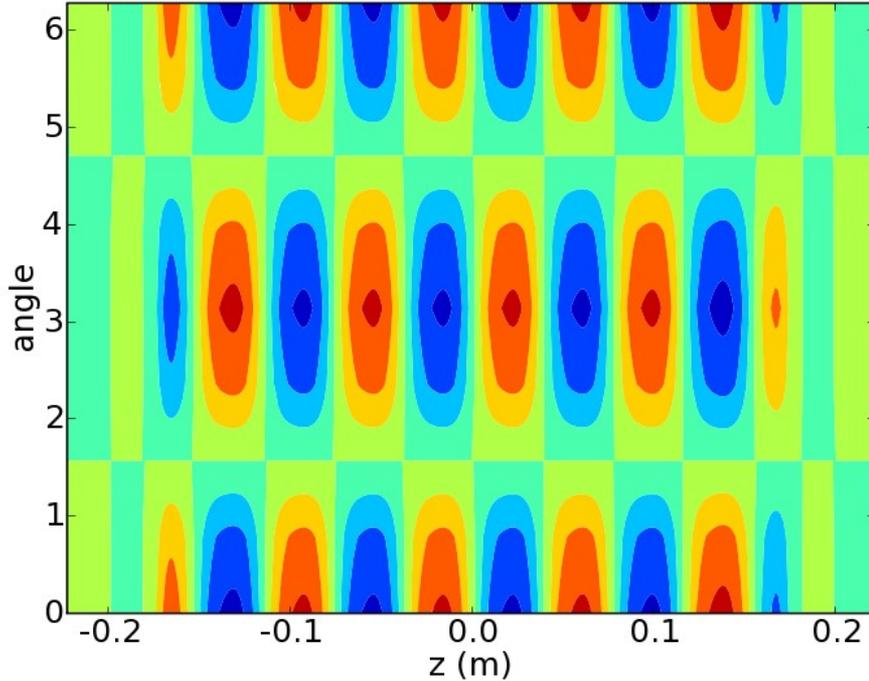


Analytic (blue) and Abell model (green) for  $E_r$  (left) and  $E_z$  (right)

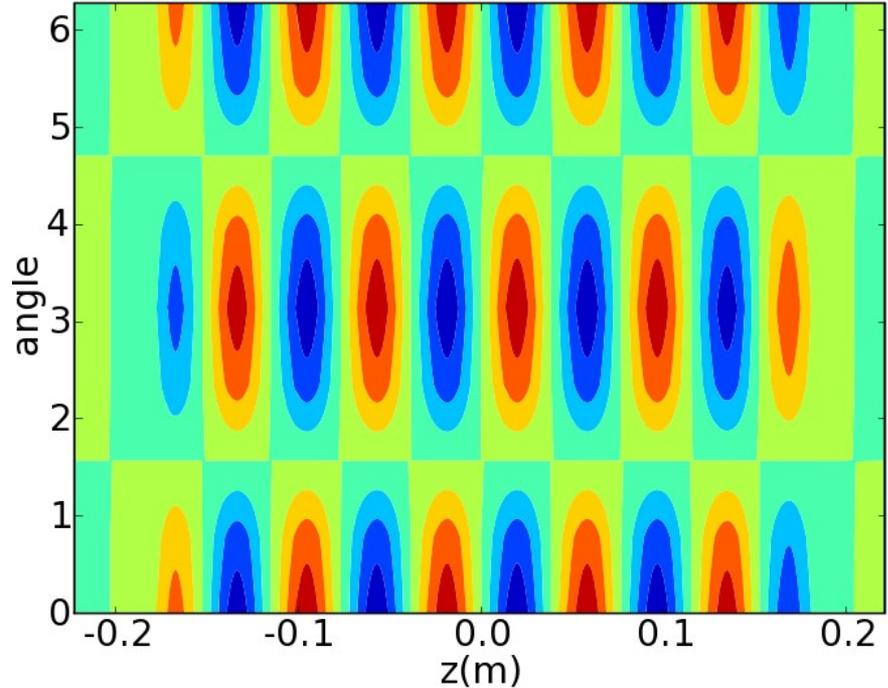
# Ex 3: Coupling to tracking codes: a nine-cell crab mode from VORPAL used as a map in MaryLie-Impact



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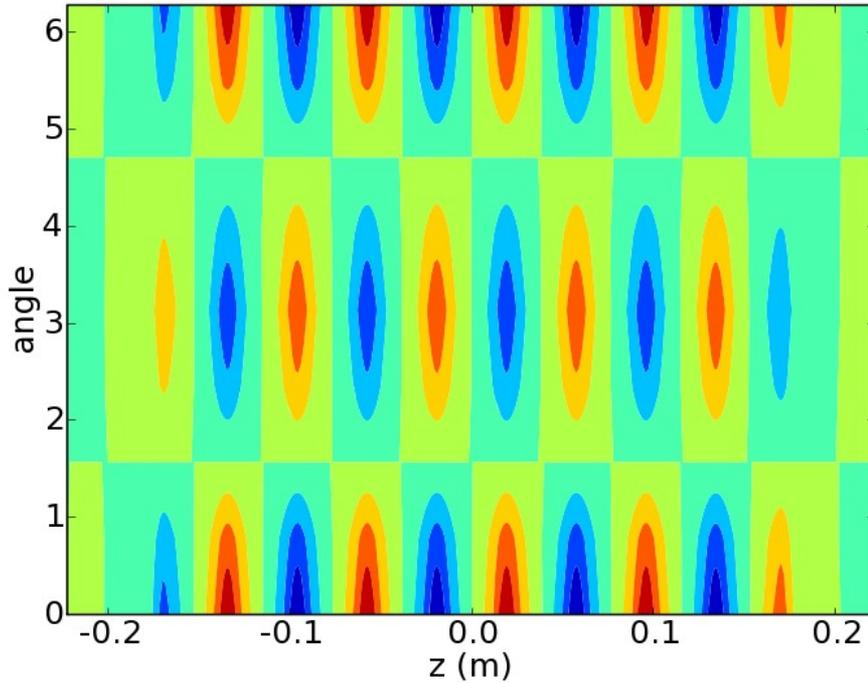


– EM simulation

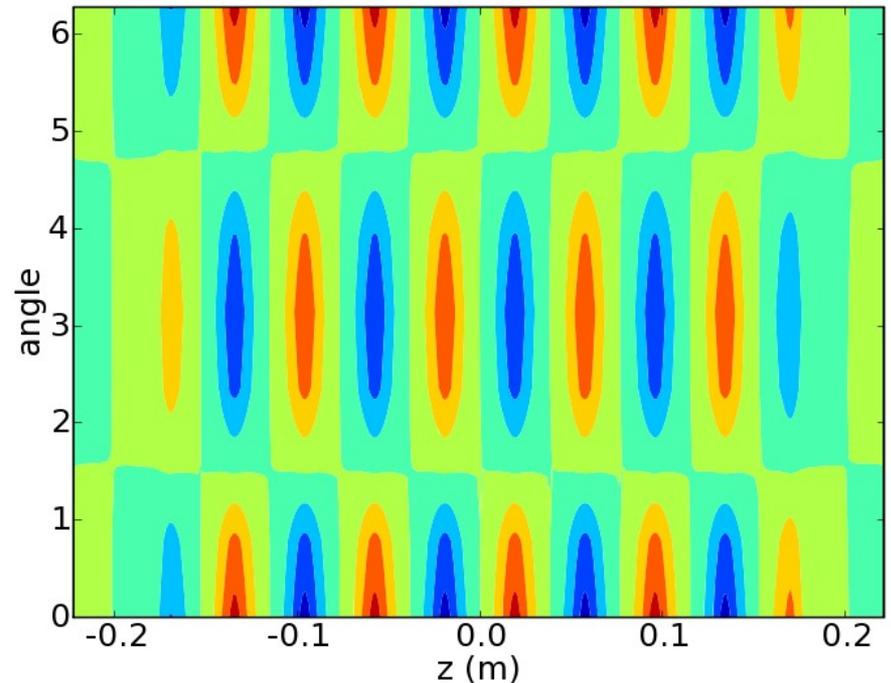


– Reconstructed

# Ex 3: Coupling to tracking codes: a nine-cell crab mode from VORPAL used as a map in MaryLie-Impact

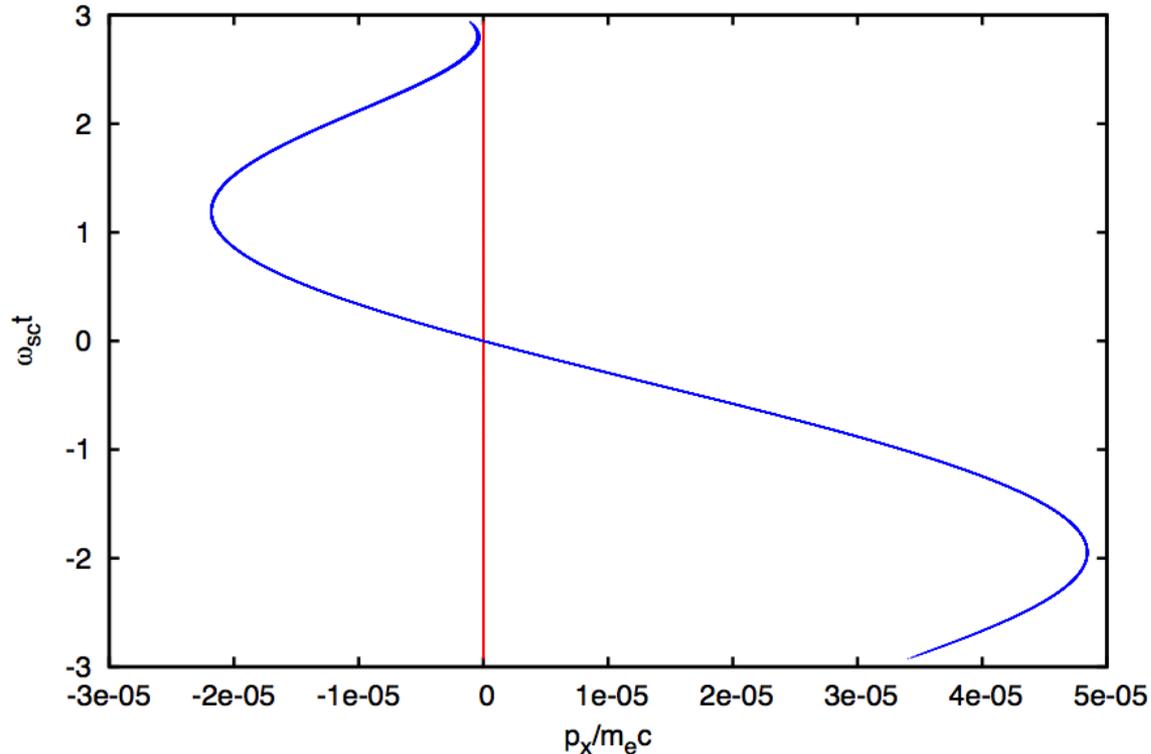


– EM simulation



– Reconstructed

## Ex 3: Coupling to tracking codes: a nine-cell crab mode from VORPAL used as a map in MaryLie-Impact



The  $p_x$ - $t$  phase space of the electron bunch at the entrance (red) and exit (blue) of the crab cavity map. Particles entering at different phases are given different transverse kicks by the map, as expected for the crab cavity.

## Coming soon: validation of map results, misalignments, HOMs



- Are the mapping results using the crab maps correct? We think so, but need to compare with detailed tracking (can use VORPAL or PIC3P to step particles through the cavity).
  - A particularly powerful application: misalignments: one can simply offset and/or rotate the axes when extracting data from the EM simulation
  - One can model HOMs too. Create a map for each mode (fundamental, HOM1, HOM2) and apply the map for each mode. Needs validation.
  - Project scheduled to finish summer 2009, but likely to continue at least to the end of 2009.
  - Ideas for applications, test problems, etc, are welcome!
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